

Math 544 Exam 4 SP 2002

PRINT Your Name: _____

There are 9 problems on 4 pages. Problem 3 is worth 10 points. Each of the other problems is worth 5 points. SHOW your work. **CIRCLE** your answer. **CHECK** your answer whenever possible. **No Calculators.**

1. Define "eigenvalue". Use complete sentences.

Let A be a square matrix. The number λ is an eigenvalue of A if there exists a non-zero vector v with $Av = \lambda v$.

2. Define "dimension". Use complete sentences.

If V is a vector space with a basis which consists of n vectors, then the dimension of V is n .

3. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 7 & 4 \\ -3 & -1 \end{bmatrix}$.

$$\det(A - \lambda I) = \det \begin{pmatrix} 7-\lambda & 4 \\ -3 & -1-\lambda \end{pmatrix} = (7-\lambda)(-1-\lambda) + 12 = \lambda^2 - 6\lambda - 7 + 12 = \lambda^2 - 6\lambda + 5$$

$$= (\lambda - 5)(\lambda - 1)$$

The e space for $\lambda = 5$ $A - 5I = \begin{bmatrix} 2 & 4 \\ -3 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ is a basis for the e. space belonging to $\lambda = 5$

$$\frac{v}{v} \begin{pmatrix} 7 & 4 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -14+4 \\ 6-1 \end{pmatrix} = \begin{pmatrix} -10 \\ 5 \end{pmatrix} = 5 \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

The e space for $\lambda = 1$ $A - I = \begin{bmatrix} 6 & 4 \\ 3 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{2}{3} \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ is a basis for the e space belonging to $\lambda = 1$

$$\frac{v}{v} \begin{pmatrix} 7 & 4 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 14-12 \\ -6+3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$