6. Let W be the subspace of $\mathbb{R}^4$ ~~whose basis is~~ which is spanned by

$$w_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \quad w_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

Find an orthogonal set which forms a basis for W.

use Gram-Schmidt.

Let $V_1 = W_1$

Let $V_2 = W_2 - \frac{V_1^T W_1}{V_1^T V_1} V_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} - \frac{2}{1}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

Let $V_3 = W_3 - \frac{V_1^T W_3}{V_1^T V_1} V_1 - \frac{V_2^T W_3}{V_2^T V_2} V_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} - 0 - \frac{3}{3}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

so W_3 is already in the span of w_1, w_2

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

7. Is

$$\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \, x_1 x_2 = x_3 \right\}$$

a vector space? If so, explain why. If not, give an example to show that one of the rules of vector space fails to hold.

NO $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in$ this set but $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \notin$ this set

$1 \cdot 1 = 1$ $\qquad$ $2 \cdot 2 \neq 2$