PRINT Your Name:_____
There are 9 problems on 4 pages. Problem 1 is worth 20 points. Each of the other problems is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. No Calculators.

1. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 5 \\ 2 \\ 1 \\ 4 \end{bmatrix}.$$

(It might be to your advantage to notice that the columns of A form an orthogonal set.)
(a) Find a matrix B so that BA is equal to the 3×3 identity matrix.
(b) Solve $Ax = b$.

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\text{So } B = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}}$$

(b) $Ax = b$
$BAx = Bb$
$$x = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 1 \\ 4 \end{bmatrix} = \boxed{\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}}$$

2. Define "column space". Use complete sentences.

The column space of a matrix is the vector space which is spanned by its columns.