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10. Find an orthogonal set which is a basis for the null space of $[1 \quad 2 \quad 2 \quad 1]$.

One basis is $w_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $w_2 = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $w_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$. Apply G–S.

Let $u_1 = w_1$

$$u_2 = w_2 - \frac{u_1^T w_2}{u_1^T u_1} u_1 = \begin{bmatrix} -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \frac{4}{5} \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -2 \\ -4 \\ 5 \\ 0 \end{bmatrix}$$

$$u_3 = w_3 - \frac{u_1^T w_3}{u_1^T u_1} u_1 - \frac{u_2^T w_3}{u_2^T u_2} u_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \frac{2}{5} \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \frac{\frac{1}{5}(2)}{\frac{1}{25}(4+16+25)} \frac{1}{5} \begin{bmatrix} -2 \\ -4 \\ 5 \\ 0 \end{bmatrix}$$

$$= \frac{1}{45} \left(\begin{bmatrix} -45 \\ 0 \\ 0 \\ 45 \end{bmatrix} + \begin{bmatrix} 36 \\ -18 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 8 \\ -10 \\ 0 \end{bmatrix} \right) = \frac{1}{45} \begin{bmatrix} -5 \\ -10 \\ -10 \\ 45 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} -1 \\ -2 \\ -2 \\ 9 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \frac{1}{5} \begin{bmatrix} -2 \\ -4 \\ 5 \\ 0 \end{bmatrix}, \frac{1}{9} \begin{bmatrix} -1 \\ -2 \\ -2 \\ 9 \end{bmatrix} \right\}$$