6. Solve $Ax = b$, where

$$A = \begin{bmatrix} 10 & 1 & 35 \\ 11 & -2 & 2 \\ 12 & 1 & -31 \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(Note. The columns of A form an orthogonal set.)

$Ax = b \qquad x = A^{-1}b = \begin{bmatrix} 10 & 11 & 12 \\ 1 & -2 & 1 \\ 35 & 2 & -31 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{365} & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{2190} \end{bmatrix}\begin{bmatrix} 33 \\ 0 \\ +6 \end{bmatrix}$

$\begin{bmatrix} \frac{1}{365} & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{2190} \end{bmatrix}$

$= \boxed{\begin{bmatrix} \frac{33}{365} \\ \frac{0}{6} \\ +\frac{6}{2190} \end{bmatrix}}$

7. Find all eigenvalues and eigenvectors of

$$A = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}.$$

$\det(A - \lambda I) = \det\begin{bmatrix} \frac{1}{2} - \lambda & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} - \lambda \end{bmatrix} = \left(\frac{1}{2} - \lambda\right)\left(-\frac{1}{2} - \lambda\right) - \frac{3}{4} = \lambda^2 - 1 = (\lambda - 1)(\lambda + 1)$

The evalues are $\lambda = 1, -1$

The e.vectors belonging to $\lambda = 1$ $\quad A - I = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \xrightarrow{R2 \mapsto R2 - \frac{1}{\sqrt{3}}R1} \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 \end{bmatrix}$

So $x_1 = \sqrt{3}x_2$
x_2 is arbitrary

the e.vectors belonging to $\lambda = 1$ are $\left\{ c\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} \middle| c \in \mathbb{R} \right\}$