

PRINT Your Name: \_\_\_\_\_

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. **CIRCLE** your answer. **CHECK** your answer whenever possible. **NO CALCULATORS.**

1. Define "linear transformation". A linear transformation is a function

$T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  with

- (1)  $T(v+w) = T(v) + T(w)$  for all  $v$  and  $w \in \mathbb{R}^n$   
 (2)  $T(cv) = cT(v)$  for all  $c \in \mathbb{R}$  and  $v \in \mathbb{R}^n$ .

2. Define "eigenvalue"

The number  $\lambda$  is an eigenvalue of the square matrix  $A$  if there is a vector  $v$ , with  $v \neq 0$ , such that  $Av = \lambda v$ .

3. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If  $A$  and  $B$  are  $2 \times 2$  matrices, then  $\det(A+B) = \det A + \det B$ .

False Take  $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$   $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$   $\det A = \det B = 0$  but  
 $\det(A+B) = 1$ .