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6. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 3x_2 \\ 2x_1 + 5x_2 \end{bmatrix}$.
Find a formula for T^{-1}. Check your answer.

$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ so $T^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \frac{1}{-1}\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$T^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -5x_1 + 3x_2 \\ 2x_1 - x_2 \end{bmatrix}$$

$$T^{-1}\left(T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = T^{-1}\begin{pmatrix} x_1 + 3x_2 \\ 2x_1 + 5x_2 \end{pmatrix} = \begin{bmatrix} -5(x_1 + \cancel{3x_2}) + 3(2x_1 + \cancel{5x_2}) \\ 2(\cancel{x_1} + 3x_2) - (\cancel{2x_1} + 5x_2) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \checkmark$$

7. The function $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation with $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$. Find a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$.
Check your answer.

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) - T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

T is a l.t.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 3 & 3 \end{bmatrix}$$

Checks $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \checkmark$

$A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \checkmark$