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4. Are

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \text{and} \quad v_4 = \begin{bmatrix} 3 \\ 2 \\ 8 \\ -3 \end{bmatrix}$$

linearly dependent or linearly independent? Show your work. Check your answer.

$$\begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 8 \\ 0 & 1 & 1 & -3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 5 \\ 0 & 1 & 1 & -3 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 1 & -5 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R3 \mapsto R3 - R1$ $\quad$ $R4 \to R4 - R2$, $R3 \mapsto -R3$ $\quad$ $R1 \to R1 - R3$, $R4 \mapsto R4 - R3$

$x_1 = -8x_4$
$x_2 = -2x_4$
$x_3 = 5x_4$
$x_4 = x_4$

$$-8\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - 2\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 8 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Leftarrow$ This is a non-trivial linear combination of v_1, v_2, v_3, v_4 which equals 0

v_1, v_2, v_3, v_4 are linearly dependent

5. Let $A = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 0 \\ 3 & 2 & 6 \end{bmatrix}$. Find A^{-1}. Check your answer.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 3 & 2 & 6 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 0 & 0 \\ 0 & 1 & -5 & -1 & 1 & 0 \\ 0 & 2 & -9 & -3 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 0 & 0 \\ 0 & 1 & -5 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array}\right]$$

$R2 \mapsto R2 - R1$, $R3 \mapsto R3 - 3R1$ $\quad$ $R3 \mapsto R3 - 2R2$ $\quad$ $R1 \mapsto R1 - 5R3$, $R2 \mapsto R2 + 5R3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & 10 & -5 \\ 0 & 1 & 0 & -6 & -9 & 5 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array}\right]$$

so $A^{-1} = \begin{bmatrix} 6 & 10 & -5 \\ -6 & -9 & 5 \\ -1 & -2 & 1 \end{bmatrix}$

ch $AA^{-1} = \begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 0 \\ 3 & 2 & 6 \end{bmatrix} \begin{bmatrix} 6 & 10 & -5 \\ -6 & -9 & 5 \\ -1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓