9. Let $v_1, \ldots, v_n$ be n linearly independent vectors in $\mathbb{R}^n$. Prove that $v_1, \ldots, v_n$ is a basis for $\mathbb{R}^n$.

Let A be the matrix whose columns are $v_1 \ldots v_n$. The columns of A are linearly independent; hence, A is invertible by the IMT; hence, the columns of A span $\mathbb{R}^n$ again by the IMT.

10. Let A and B be 2×2 matrices with A invertible. Does the columns space of AB have to equal the column space of B? If the answer is yes, prove it. If the answer is no, give a counterexample.

No The column space of AB has nothing to do with the column space of B.

Ex Take $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$. Notice that the column space of B is the set of all multiples of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

The product $AB = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$. The column space of AB is spanned by $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$. These two vector spaces have only the zero vector in common.