7. Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 9 \\ 3 & 6 & 6 \end{bmatrix}$. Find a basis for the null space of A. Find a basis for the column space of A.

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 9 \\ 3 & 6 & 6 \end{bmatrix} \xrightarrow[R3 \mapsto R3-3R1]{R2 \mapsto R2-2R1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & -6 \end{bmatrix} \xrightarrow[R3 \mapsto R3+6R2]{R1 \mapsto R1-4R2} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

If $Ax=0$ then $x_1 = -2x_2$
$x_2 = x_2$
$x_3 = 0$

So $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ is a basis for the null space of A

columns 1 and 3 of the reduced matrix have leading ones

So $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 6 \end{bmatrix}$ is a basis for the column space of A

8. Let V be the subspace of $\mathbb{R}^3$ which is spanned by

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 4 \\ 9 \\ 6 \end{bmatrix}, \quad \text{and} \quad v_4 = \begin{bmatrix} 6 \\ 13 \\ 12 \end{bmatrix}.$$

Find a basis for V.

V is the column space of $\begin{bmatrix} 1 & 2 & 4 & 6 \\ 2 & 4 & 9 & 13 \\ 3 & 6 & 6 & 12 \end{bmatrix} \xrightarrow[R3 \mapsto R3-3R1]{R2 \mapsto R2-2R1} \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -6 & -6 \end{bmatrix} \xrightarrow[R3 \mapsto R3+6R2]{R1 \mapsto R1-4R2}$

$$\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

v_1 and v_3 are a basis for V

Check the null space of A is $x_1 = -2x_2 - 2x_4$
$x_2 = x_2$
$x_3 = -x_4$
$x_4 = x_4$

So $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ are a basis for the null space of A.

so $-2v_1 + v_2 = 0$ i.e. $v_2 = 2v_1$ ✓
$2v_1 + v_3 = v_4$ ✓