4. Let $W = \{f\colon \mathbb{R} \to \mathbb{R} \mid f \text{is continuous}\}$. Is W a vector space? Explain.

Yes. The sum of 2 continuous functions is continuous and a scalar multiple of a continuous function is continuous. The zero function is continuous.

5. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the function $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ \sin y \end{bmatrix}$. Is T a linear transformation? Explain.

No $T\begin{bmatrix} 0 \\ \frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $T\begin{bmatrix} 0 \\ 2(\frac{\pi}{2}) \end{bmatrix} = T\begin{bmatrix} 0 \\ \pi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

so $2T\begin{bmatrix} 0 \\ \frac{\pi}{2} \end{bmatrix} \neq T\left(2\begin{bmatrix} 0 \\ \frac{\pi}{2} \end{bmatrix}\right)$

6. Give an example of three 2×2 matrices A, B, and C, with A not the zero matrix, and $B \neq C$, but $AB = AC$.

$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}$

we see $A \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B \neq C$, but $AB = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 8 & 8 \end{bmatrix}$

and $AC = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 8 & 8 \end{bmatrix}$