45 ~~38~~

8. Let W be the subspace of $\mathbb{R}^4$ which is spanned by

$$\begin{bmatrix}1\\3\\-1\\0\end{bmatrix}, \begin{bmatrix}2\\1\\1\\-2\end{bmatrix}, \begin{bmatrix}-1\\1\\-2\\1\end{bmatrix}, \begin{bmatrix}1\\2\\2\\2\end{bmatrix}.$$

Find a basis for W.

W is the column space of $\begin{bmatrix}1&2&-1&1\\3&1&1&2\\-1&1&-2&2\\0&-2&1&2\end{bmatrix}$

$R2 \mapsto R2-3R1$, $R3 \mapsto R3+R1$ $\begin{bmatrix}1&2&-1&1\\0&-5&4&-1\\0&3&-3&3\\0&-2&1&2\end{bmatrix}$ $R2 \to R3$ $\begin{bmatrix}1&2&-1&1\\0&3&-3&3\\0&-5&4&-1\\0&-2&1&2\end{bmatrix}$ $R2 \mapsto \frac{1}{3}R2$ $\begin{bmatrix}1&2&-1&1\\0&1&-1&1\\0&-5&4&-1\\0&-2&1&2\end{bmatrix}$ $R3 \mapsto R3+5R2$, $R4 \mapsto R4+2R2$ $\begin{bmatrix}1&2&-1&1\\0&1&-1&1\\0&0&-1&+4\\0&0&-1&4\end{bmatrix}$

$R4 \mapsto R4-R3$ $\begin{bmatrix}1&2&-1&1\\0&1&-1&1\\0&0&-1&4\\0&0&0&0\end{bmatrix}$

leading ones

basis for W $\begin{bmatrix}1\\3\\-1\\0\end{bmatrix}, \begin{bmatrix}2\\1\\1\\-2\end{bmatrix}, \begin{bmatrix}-1\\1\\-2\\1\end{bmatrix}.$

9. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If U and V are subspaces of $\mathbb{R}^n$, then the union of U and V is also a subspace of $\mathbb{R}^n$.

False Let $U = \left\{ \begin{bmatrix}a\\0\end{bmatrix} \middle| a \in \mathbb{R} \right\}$ and $V = \left\{ \begin{bmatrix}0\\a\end{bmatrix} \middle| a \in \mathbb{R} \right\}$.

we see that $\begin{bmatrix}1\\0\end{bmatrix} \in U \cup V$ $\begin{bmatrix}0\\1\end{bmatrix} \in U \cup V$ but

$\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} \notin U \cup V$