

5. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If  $A$  and  $B$  are  $2 \times 2$  matrices, then

the null space of  $A \cap$  the null space of  $B \subseteq$  the null space of  $A + B$ .

True If  $x$  is a vector on the left side, then  $Ax=0$  and  $Bx=0$ . It follows that  $(A+B)x = Ax+Bx = 0+0 = 0$ . So  $x$  is also on the right side.

6. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If  $A$  and  $B$  are  $2 \times 2$  nonsingular matrices, then  $A + B$  is a nonsingular matrix.

False Let  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ . We see that  $A$  and  $B$  are nonsingular but  $A+B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$  which is singular.

7. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If  $A$  and  $B$  are  $2 \times 2$  nonsingular matrices, then  $AB$  is a nonsingular matrix.

True If  $A$  and  $B$  are nonsingular then they each have inverses. It follows that  $AB$  also has an inverse, namely  $B^{-1}A^{-1}$ .