$\boxed{43}$ ~~33~~

3. Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & -1 \\ 2 & 2 & 0 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

(a) Find a basis for the null space of A.
(b) Find a basis for the column space of A.
(c) Find a basis for the row space of A.

$R2 \mapsto R2 - 2R1$
$R3 \mapsto R3 - 2R1$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R3 \mapsto R3 + 2R2$
$R4 \mapsto R4 - R2$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & -\cancel{1} & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ \cancel{0} & 0 & 0 & 0 \end{bmatrix}$$

leading ones.

$R1 \mapsto R1 - 2R2$

The null space is
$x_1 = \cancel{1} x_3 - 2x_4$
$x_2 = -x_3 + x_4$
$x_3 = x_3$
$x_4 = x_4$

a basis for the null space is

$$\begin{bmatrix} \cancel{1} \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

(a)

a basis ~~for the~~ col. space is

$$\begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 2 \\ 1 \end{bmatrix}$$

(b)

a basis for the row space is

$[1 \quad 0 \quad -1 \quad 2]$,
$[0 \quad 1 \quad 1 \quad -1]$

(c)

$\frac{20}{}$

4. Define "basis". Use complete sentences.

The vectors $v_1, \ldots v_p$ are a basis for the vector space V if $v_1 \ldots v_p$ span V and are linearly independent