6. (10 points) Define "null space".

The <u>null space</u> of the matrix A is the set of all column vectors x with $Ax = 0$

7. (10 points) True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the column space of BA is equal to the column space of B.

True

col sp BA ⊆ col sp B:

If $x \in$ col sp. BA then $x = BA\,y$ for some y, so $x = B(Ay)$
Thus $x \in$ col space B.

If

Col sp B ⊆ col sp BA:

If $x \in$ col sp. B, then $x = By$ for some y, so $x = BA(A^{-1}y)$
Thus $x \in$ col sp. BA

8. (10 points) True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the null space of BA is equal to the null space of B.

False

$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

the null space of $B = \left\{ \begin{bmatrix} a \\ 0 \end{bmatrix} \middle| a \in \mathbb{R} \right\}$

the null space of $BA = \left\{ \begin{bmatrix} 0 \\ a \end{bmatrix} \middle| a \in \mathbb{R} \right\}$