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3. (10 points) Define "basis".

The vectors $v_1, \dots, v_p$ are a <u>basis</u> for the vector space V if

1) $v_1, \dots, v_p$ are linearly independent and
2) $v_1, \dots, v_p$ span V.

4. (10 points) True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the column space of AB is equal to the column space of B.

FALSE $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $AB = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

The column space of $B = \left\{ \begin{bmatrix} a \\ 0 \end{bmatrix} \middle| a \in \mathbb{R} \right\}$

The column space of $AB = \left\{ \begin{bmatrix} 0 \\ a \end{bmatrix} \middle| a \in \mathbb{R} \right\}$

5. (10 points) True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrcies with A non-singular, then the null space of AB is equal to the null space of B.

True <u>The null space of $B \subseteq$ null space AB always:</u>
If $x \in$ null sp. B, then $Bx = 0$ so $ABx = 0$ so $x \in$ null space A

<u>For us null space $AB \subseteq$ null space B:</u>
If $x \in$ null space AB, then $ABx = 0$ multiply both sides on the left by A^{-1} so $A^{-1}ABx = A^{-1}0$ i.e. $BX = 0$
So $x \in$ null space B