

6. The function $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is a linear transformation with $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$. Find a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

7. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.)

False

Let A , B , and C be 2×2 matrices with A not equal to the zero matrix. If $AB = AC$, then $B = C$.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 4 \\ -1 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 6 & 8 \\ -3 & -4 \end{bmatrix}$$

Observe that A is not the zero matrix

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = AC$$

$$\text{But } B \neq C$$