4. True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly independent vectors in $\mathbb{R}^4$.

False Let $T: \mathbb{R}^4 \to \mathbb{R}^4$ be $T(v) = 0$ for all $v \in \mathbb{R}^4$. It is clear that T is a linear transformation. Let $v_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$, $v_2 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$, $v_3 = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}$. We see v_1, v_2, v_3 are l.i But $T(v_1) = T(v_2) = T(v_3) = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$ So $T(v_1), T(v_2), T(v_3)$ are ~~is~~ not l.i.

5. True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly dependent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly dependent vectors in $\mathbb{R}^4$.

True we are given numbers $c_1, c_2, c_3 \in \mathbb{R}$ with $c_1v_1 + c_2v_2 + c_3v_3 = 0$ but not all of the c's are zero.

Apply T to both sides to get

$T(c_1v_1 + c_2v_2 + c_3v_3) = T(0) = 0$

$c_1 T(v_1) + c_2 T(v_2) + c_3 T(v_3)$

with some c_i still not zero.

Thus $T(v_1), T(v_2), T(v_3)$ are linearly dependent.