

PRINT Your Name: _____

There are 10 problems on 5 pages. Each problem is worth 5 points. SHOW your work. **CIRCLE** your answer. **CHECK** your answer whenever possible. **No Calculators.**

1. Find the general solution of the following system of linear equations.

$$x_1 + 2x_2 + 2x_3 + x_4 = 2$$

$$x_1 + 2x_2 + 3x_3 + 2x_4 = 3.$$

Also find **three** particular solutions of this system of equations. **Be sure to check** that all three of your particular solutions really satisfy the original system of linear equations.

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 2 \\ 1 & 2 & 3 & 2 & 3 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 2 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{array} \right]$$

The general solution is $x_1 = -2x_2 + x_4$

$$x_2 = x_2$$

$$x_3 = 1 - x_4$$

$$x_4 = x_4$$

Some particular solutions are

$$(x_2 = x_4 = 0) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(x_2 = 1, x_4 = 0) \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$(x_2 = 0, x_4 = 1) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$0 + 0 + 2 + 0 = 2 \checkmark$$

$$0 + 0 + 3 + 0 = 3 \checkmark$$

$$-2 + 2 + 2 = 2 \checkmark$$

$$-2 + 2 + 3 + 0 = 3 \checkmark$$

$$1 + 0 + 0 + 1 = 2 \checkmark$$

$$1 + 0 + 0 + 2 = 3 \checkmark$$