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544 Exam 2 1998

1. The vectors $v_1, \ldots, v_p$ are <u>linearly dependent</u> if there exist numbers $c_1, \ldots, c_p$, not all zero, with $c_1 v_1 + \cdots + c_p v_p = 0$.

2. The $n \times n$ matrix A is <u>non-singular</u> if the only column vector x with $Ax = 0$ is $x = 0$.

3. True. If $x \in$ nullspace $A \cap$ nullspace of B, then $Ax = 0$ and $Bx = 0$. Thus $(A+B)x = Ax + Bx = 0 + 0 = 0$. We conclude that $x \in$ the nullspace of $A+B$.

4. False. $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are singular matrices but $A+B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a non-singular matrix.

5. $\left[\begin{array}{cc|c} 2 & 4 & 9 \\ 3 & 6 & 5 \end{array}\right] \xrightarrow{R1 \mapsto \frac{1}{2}R1} \left[\begin{array}{cc|c} 1 & 2 & \frac{9}{2} \\ 3 & 6 & 5 \end{array}\right] \xrightarrow{R2 \mapsto R2 - 3R1} \left[\begin{array}{cc|c} 1 & 2 & \frac{9}{2} \\ 0 & 0 & 5 - \frac{39}{2} \end{array}\right]$.

If $5 - \frac{39}{2} = 0$, then this system of equations has infinitely many solutions. If $5 - \frac{39}{2} \neq 0$, then the system has no solution.

The system has no solution <u>unless</u> $\frac{10}{3} = 9$.

6. $\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 1 & 0 & -1 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R3 \mapsto R3 - R1} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 0 & -1 & -1 & 1 & 2 & -1 \end{array}\right] \xrightarrow{\substack{R1 \mapsto R1 - R2 \\ R3 \mapsto R1 + R3}} \left[\begin{array}{ccccc|c} 1 & 0 & -2 & -1 & -4 & 0 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 0 & 0 & 1 & 2 & 5 & 0 \end{array}\right] \xrightarrow{\substack{R1 \mapsto R1 + 2R3 \\ R2 \mapsto R2 - 2R3}}$

$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 3 & 6 & 0 \\ 0 & 1 & 0 & -3 & -7 & 1 \\ 0 & 0 & 1 & 2 & 5 & 0 \end{array}\right]$

$x_1 = -3x_4 - 6x_5$

$x_2 = 1 + 3x_4 + 7x_5$

$x_3 = -2x_4 - 5x_5$

x_4 and x_5 are arbitrary