24 19

Problems 9 and 10 both use the matrix

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 1 & 3 & 1 & 6 & 0 \\ 1 & 3 & 1 & 6 & 1 \\ 2 & 6 & 1 & 8 & 1 \end{bmatrix}.$$

9. Find a basis for the null space of A.

10. Find a basis for the column space of A.

$R2 \mapsto R2 - R1$
$R3 \mapsto R3 - R1$
$R4 \mapsto R4 - 2R1$

$$\begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 4 & 1 \end{bmatrix}$$

$R3 \mapsto R3 - R2$
$R4 \mapsto R4 - R2$

$$\begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R4 \mapsto R4 - R3$

$$\begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

nullspace x_2 and x_4 are arbitrary

$$\begin{aligned} x_1 &= -3x_2 - 2x_4 \\ x_2 &= x_2 \\ x_3 &= -4x_4 \\ x_4 &= x_4 \\ x_5 &= 0 \end{aligned}$$

so the nullspace is $\left\{ x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 0 \\ -4 \\ 1 \\ 0 \end{bmatrix} \;\middle|\; x_2, x_4 \in \mathbb{R} \right\}$

a basis for the nullspace is $\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -4 \\ 1 \\ 0 \end{bmatrix}$

9

A basis for the column space is

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

10

col 1 col 3 col 5