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7. Is $v = \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix}$ in the column space of $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}$? Justify your answer.

Yes $\begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix} - 1 \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$

One sees this by solving $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 4 & 5 & 3 \\ 6 & 7 & 5 \end{array}\right]$ $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -3 & 3 \\ 0 & -5 & 5 \end{array}\right]$ $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & -5 & 5 \end{array}\right]$

$R2 \mapsto R2 - 4R1$
$R3 \mapsto R3 - 6R1$

$R2 \mapsto -\frac{1}{3} R2$

$R_3 \mapsto R_3 - 5R_2$
$R1 \mapsto R1 - 2R2$

$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$ Thus $c_1 = 2$ and $c_2 = -1$

8. Let A be a fixed 2×3 matrix. Let $V = \{ v \in \mathbb{R}^2 \mid v = Ax \text{ for some } x \in \mathbb{R}^3 \}$.
Is V a subspace of $\mathbb{R}^2$? Justify your answer.

Yes V is the column space of A.

or if you like:

$0 \in V$ because $0 = A0$;

closed under + if v_1 and v_2 are in V, then $v_i = Ax_i$ for some $x_1, x_2 \in \mathbb{R}$

So $v_1 + v_2 = A(x_1 + x_2) \in V$

closed under sc. mult If $v \in V$ and $c \in \mathbb{R}$, then $v = Ax$ for some $x \in \mathbb{R}^3$

so $cv = A(cx) \in V$.