544 Exam 2 1997

21 B

PRINT Your Name:________________________

There are 10 problems on 4 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

1. Define "linearly independent".

The vectors $v_1, \dots, v_p$ in $\mathbb{R}^n$ are linearly independent if the only numbers $c_1, \dots, c_p$ in $\mathbb{R}$ with $c_1 v_1 + \dots + c_p v_p = 0$ are $c_1 = c_2 = \dots = c_p = 0$.

2. Define "nonsingular".

The $n \times n$ matrix A is nonsingular if the only vector $x \in \mathbb{R}^n$ with $Ax = 0$ is $x = 0$.

3. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are $n \times n$ matrices, then

the null space of $A + B \subseteq$ the null space of $A \cap$ the null space of B.

False. Take $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ so $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is in the null space of $A+B$, but

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is not in the null space of A

(also $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is not in the null space of B).