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5. Find the general solution of the following system of linear equations:

$$\begin{array}{rcrcrcl} x_1 & + & x_2 & & & - & x_5 = 1 \\ & & x_2 & +2x_3 & + x_4 & +3x_5 & = 1 \\ x_1 & & & - x_3 & + x_4 & + x_5 & = 0. \end{array}$$

Also find **three** particular solutions of this system of equations. **Be sure to check** that all three of your particular solutions really satisfy the original system of linear equations.

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 1 & 0 & -1 & 1 & 1 & 0 \end{array}\right] \quad R3 \mapsto R3 - R1$$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 0 & -1 & -1 & 1 & 2 & -1 \end{array}\right] \quad \begin{array}{l} R3 \mapsto R3 + R2 \\ R1 \mapsto R1 - R2 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -2 & -1 & -4 & 0 \\ 0 & 1 & 2 & 1 & 3 & 1 \\ 0 & 0 & 1 & 2 & 5 & 0 \end{array}\right] \quad \begin{array}{l} R2 \mapsto R2 - 2R3 \\ R1 \mapsto R1 + 2R3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 3 & 6 & 0 \\ 0 & 1 & 0 & -3 & -7 & 1 \\ 0 & 0 & 1 & 2 & 5 & 0 \end{array}\right]$$

So

$$\begin{array}{l} x_1 = -3x_4 - 6x_5 \\ x_2 = 1 + 3x_4 + 7x_5 \\ x_3 = -2x_4 - 5x_5 \\ x_4 = x_4 \\ x_5 = x_5 \end{array}$$

General solution

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -3 \\ 4 \\ -2 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -6 \\ 8 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

are particular solutions

check

$$\begin{array}{l|l|l} 1 = 1 & -3 + 4 = 1 & -6 + 8 - 1 = 1 \\ 1 = 1 & 4 - 4 + 1 = 1 & 8 - 10 + 3 = 1 \\ 0 = 0 & -3 + 2 + 1 = 0 & -6 + 5 + 1 = 0 \end{array}$$