10. True or False. (If true, explain why or give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ and v_4 is a vector in $\mathbb{R}^4$ which is not a linear combination of v_1, v_2, and v_3, then v_1, v_2, v_3, v_4 are linearly independent vectors in $\mathbb{R}^4$.

(**)

True suppose $c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 = 0.$ If $c_4 \neq 0$ then $v_4 = -\frac{c_1}{c_4} v_1 - \frac{c_2}{c_4} v_2 - \frac{c_3}{c_4} v_3$ and we have expressed v_4 as a linear combination of v_1, v_2, v_3, which is not possible according to the hypothesis. It follows that c_4 must be zero. Now we have $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$. The hypothesis also tells us that v_1, v_2, v_3 are linearly independent. ~~We~~ Thus $c_1 = c_2 = c_3 = 0$. We have shown that the only numbers c_1, c_2, c_3, and c_4 which satisfy (**) are $c_1 = c_2 = c_3 = c_4 = 0$. It follows that v_1, v_2, v_3, v_4 are linearly independent.