8. Fill in the blank with an inequality involving m and p and then prove ~~proof~~ the result. Let $v_1, \dots, v_p$ be vectors in $\mathbb{R}^m$. If $m < p$, then $v_1, \dots, v_p$ are linearly dependent. $\leftarrow \circledast$

The equation $c_1 v_1 + \dots + c_p v_p = 0$ corresponds to a homogeneous system of linear equations with m equations and p variables. This system of equations has at least one solution because $c_1 = c_2 = \dots = c_p = 0$ is a solution. When we put the matrix of coefficients into Row Reduced Echelon Form we see that the number of leading ones $\leq$ the number of rows $<$ the number of columns. It follows that some column does not contain a leading one. The corresponding variable may take any value; hence $\circledast$ has more than one solution and $v_1, \dots, v_p$ are linearly dependent.

9. True or False. (If true, explain why or give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly independent vectors in $\mathbb{R}^3$.

<u>False</u> Consider the function $T: \mathbb{R}^4 \to \mathbb{R}^3$ which is given by $T(v) = 0$ for all $v \in \mathbb{R}^4$. This function is a linear transformation. If $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, then v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ but $T(v_1) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $T(v_2) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $T(v_3) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ are linearly dependent vectors in $\mathbb{R}^3$.