

5. Compute

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2+9 \\ 1+12 \end{bmatrix} = \begin{bmatrix} 11 \\ 13 \end{bmatrix}$$

6. Find scalars a_1 and a_2 so that $a_1 r + a_2 s = t$, where

$$r = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad s = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \text{and} \quad t = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$

Solve $\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 3 & 4 \end{array} \right] \xrightarrow{R_2 \rightarrow \frac{1}{3}R_2} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & \frac{4}{3} \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left[\begin{array}{cc|c} 1 & 0 & -\frac{5}{3} \\ 0 & 1 & \frac{4}{3} \end{array} \right]$

So $\begin{cases} a_1 = -\frac{5}{3} \\ a_2 = \frac{4}{3} \end{cases}$

7. Find x so that $x^T a = 6$ and $x^T b = 2$, where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

Solve $\begin{cases} x_1 + 2x_2 = 6 \\ 3x_1 + 4x_2 = 2 \end{cases} \quad \left[\begin{array}{cc|c} 1 & 2 & 6 \\ 3 & 4 & 2 \end{array} \right] \xrightarrow{R_2 \rightarrow R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & 2 & 6 \\ 0 & -2 & -16 \end{array} \right] \xrightarrow{R_2 \rightarrow -\frac{1}{2}R_2}$

$\left[\begin{array}{cc|c} 1 & 2 & 6 \\ 0 & 1 & 8 \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left[\begin{array}{cc|c} 1 & 0 & -10 \\ 0 & 1 & 8 \end{array} \right]$

So $x = \begin{bmatrix} -10 \\ 8 \end{bmatrix}$