PRINT Your Name:_____

There are 9 problems on 4 pages. Problem 1 is worth 12 points. Each of the other problems is worth 11 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

1. Solve the following system of equations:

$$\begin{aligned}
x_1 &+ x_2 & & & - x_5 &= 1 \\
& x_2 &+ 2x_3 &+ x_4 &+ 3x_5 &= 1 \\
x_1 & &- x_3 &+ x_4 &+ x_5 &= 0
\end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & | & 1 \\ 0 & 1 & 2 & 1 & 3 & | & 1 \\ 1 & 0 & -1 & 1 & 1 & | & 0 \end{bmatrix} \begin{matrix} \\ \\ R3 \mapsto R3-R1 \\ R1 \mapsto R1-R2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & -1 & -4 & | & 0 \\ 0 & 1 & 2 & 1 & 3 & | & 1 \\ 0 & -1 & -1 & 1 & 2 & | & -1 \end{bmatrix} \begin{matrix} \\ \\ R3 \mapsto R3+R2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & -1 & -4 & | & 0 \\ 0 & 1 & 2 & 1 & 3 & | & 1 \\ 0 & 0 & 1 & 2 & 5 & | & 0 \end{bmatrix} \begin{matrix} \\ R2 \mapsto R2-2R3 \\ R1 \mapsto R1+2R3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 6 & | & 0 \\ 0 & 1 & 0 & -3 & -7 & | & 1 \\ 0 & 0 & 1 & 2 & 5 & | & 0 \end{bmatrix}$$

$$\boxed{\begin{aligned}
x_1 &= & -3x_4 - 6x_5 \\
x_2 &= 1 &+ 3x_4 + 7x_5 \\
x_3 &= & -2x_4 - 5x_5 \\
x_4 &= & x_4 \\
x_5 &= & x_5
\end{aligned}}$$

2. Find all values of a for which the following system has no solution:

$$\begin{aligned}
x_1 &+ 2x_2 &= -3 \\
ax_1 &- 2x_2 &= 5
\end{aligned}$$

$$\begin{bmatrix} 1 & 2 & | & -3 \\ a & -2 & | & 5 \end{bmatrix} \begin{matrix} \\ R2 \mapsto R2-aR1 \end{matrix} \begin{bmatrix} 1 & 2 & | & -3 \\ 0 & -2-2a & | & 5+3a \end{bmatrix}$$

If $-2-2a \neq 0$ then the system of equations has a unique solution. If $-2-2a=0$, then $-2=2a$ $a=-1$ and the bottom row is $[0 \ 0 | 2]$. The corresponding equation has no solution.

$\boxed{\text{The system of equations has no solution for } a=-1. \text{ It has a unique solution for all other } a.}$