7. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 symmetric matrices, then AB is a symmetric matrix.

(False) $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ These are symmetric

$AB = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ This is not symmetric.

8. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 nonsingular matrices, then AB is a nonsingular matrix.

(True)   A matrix is nonsingular if and only if it has an inverse. If A and B are non-singular, then let A^{-1} be the inverse of A and B^{-1} be the inverse of B. We show that $B^{-1}A^{-1}$ is the inverse of AB.

Indeed, $(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}B = I$

$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AA^{-1} = I$.

Thus AB has an inverse. It follows that AB is nonsingular.