PRINT Your Name:________________________________

Quiz for June 14, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** Write in complete sentences. Express your work in a neat and coherent manner.

The Question: Suppose $V_1 \subseteq V_2 \subseteq V_3$ are vector spaces and v_1, v_2, v_3, v_4 are vectors in V_3 which form a basis for V_3. Suppose further, that v_1, v_2, v_3 are in V_2 and $v_4 \notin V_2$. Suppose v_1, v_2 are in V_1 and $v_3 \notin V_1$. Do you have enough information to know the dimension of V_1. **Explain very thoroghly.**

The Solution: You proved on yesterday's Quiz that if $U \subseteq W$ are finite dimensional vector spaces with $U \neq W$, then $\dim U < \dim W$. We will use this fact twice in the present problem. We will also use the fact that if r linearly independent vectors live in a vector space U, then $\dim U \geq r$.

The vector space V_3 has dimension 4 because it has a basis with four vectors. The vector space V_2 is a proper subspace of V_3 because v_4 is in V_3, but not in V_2. It follows that the dimension of V_2 must be less than 4. On the other hand, the vectors v_1, v_2, v_3 are linearly independent vectors in V_2; so $\dim V_2 \geq 3$. We have shown that $\dim V_2$ must equal 3. The vector space V_1 is a proper subspace of V_2; hence $\dim V_1 \leq 2$. We have exhibited 2 linearly independent vectors in V_2; thus, $\dim V_2 \geq 2$; and in fact, $\boxed{\dim V_1 \text{ must equal } 2}$.