PRINT Your Name:________________________________

Quiz for June 13, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** Write in complete sentences. Express your work in a neat and coherent manner.

The Question: Let V and W be subspaces of $\mathbb{R}^n$ with $V \subseteq W$.

(a) Does the dimension of V have to be $\leq$ the dimension of W? If yes, then give a complete, correct, proof. If no, then give an explicit example.

(b) Suppose $\dim V = \dim W$. Does V have to equal W? If yes, then give a complete, correct, proof. If no, then give an explicit example.

The Solution: The answer to (a) is: YES. A basis for V is a linearly independent set in W. Every linearly independent set in W is contained in a basis for W, according to

Theorem 2. If V is a subspace of $\mathbb{R}^n$, then every linearly independent subset in V is part of a basis for V.

It follows that the dimension of V, which is the number of vectors in a basis for V, is less than or equal to the dimension of W, which is the number of vectors in a basis for W.

The answer to (b) is: YES. Let $v_1, \dots, v_p$ be a basis for V. Part (a) shows that $v_1, \dots, v_p$ is part of a basis for W. However every basis for W has p vectors according to

Theorem 1. If V is a subspace of $\mathbb{R}^n$, then every basis for V has the same number of vectors.

So $v_1, \dots, v_p$ are already a basis for W. In particular $v_1, \dots, v_p$ span W. Every element in W is automatically also in V. The sets V and W are equal.