Math 544, Exam 3, Summer 2007

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

Please leave room in the upper left corner for the staple.

There are **6** problems **on TWO sides**. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

You should **KEEP** this copy of your exam.

I will post the solutions on my website sometime after 3:15 today.

1. (7 points) Let $V = \{p(x) \in \mathcal{P}_3 \mid \int_0^1 p(x)dx = 0\}$. Is V a vector space? If yes, then find a basis for V. If no, then show why not? (Recall that $\mathcal{P}_3$ is the vector space of polynomials of degree less than or equal to 3.)

2. (7 points) Let V be the set of singular 2×2 matrices. Is V a vector space? If yes, then find a basis for V. If no, then show why not?

3. (7 points) Let $U \subseteq V$ be vector spaces. Suppose that v_1, v_2, v_3, v_4 is a basis for V. Suppose further that v_1 and v_2 are in U, but v_3 and v_4 are not in U. What is the dimension of U? Explain your answer **VERY THOROUGHLY.**

4. (7 points) Let A be a 3×5 matrix. Suppose that z_1, z_2, x_1, and x_2 are vectors in $\mathbb{R}^5$ and y_1 and y_2 are vectors in $\mathbb{R}^3$. Suppose further that $Az_1 = 0$, $Az_2 = 0$, $Ax_1 = y_1$, and $Ax_2 = y_2$. Suppose finally, that z_1 and z_2 are linearly independent, and that y_1 and y_2 are linearly independent. Do z_1, z_2, x_1, x_2 have to be linearly independent? If yes, give a complete proof. If no, give a counter example.

5. (11 points) In this problem, if M is a matrix, then let $\mathcal{I}(M)$ denote the column space of M. Let A and B be $n \times n$ matrices. Answer each question. If the answer is yes, then give a proof. If the answer is no, then give a counter example.
   (a) Is $\mathcal{I}(A)$ always a subset of $\mathcal{I}(AB)$?
   (b) Is $\mathcal{I}(B)$ always a subset of $\mathcal{I}(AB)$?
   (c) Is $\mathcal{I}(AB)$ always a subset of $\mathcal{I}(A)$?
   (d) Is $\mathcal{I}(AB)$ always a subset of $\mathcal{I}(B)$?
   (e) Suppose B is non-singular. Is $\mathcal{I}(A)$ always a subset of $\mathcal{I}(AB)$?
   (f) Suppose B is non-singular. Is $\mathcal{I}(B)$ always a subset of $\mathcal{I}(AB)$?
   (g) Suppose B is non-singular. Is $\mathcal{I}(AB)$ always a subset of $\mathcal{I}(A)$?
   (h) Suppose B is non-singular. Is $\mathcal{I}(AB)$ always a subset of $\mathcal{I}(B)$?

6. (11 points) Let $A = \begin{bmatrix} 1 & 7 & 8 & 5 & 1 & 8 & 7 & 2 & 11 \\ 1 & 7 & 8 & 5 & 2 & 14 & 11 & 2 & 11 \\ 1 & 7 & 8 & 5 & 2 & 14 & 11 & 3 & 13 \\ 3 & 21 & 24 & 15 & 5 & 36 & 29 & 7 & 35 \end{bmatrix}$. Find a basis for the null space of A. Find a basis for the column space of A. Find a basis for the row space of A. Express each column of A as a linear combination of the basis you have chosen for the column space of A. Express each row of A as a linear combination of the basis you have chosen for the row space of A.