**Math 544, Exam 2, Summer 2007**

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

**Please leave room in the upper left corner for the staple.**

There are **9** problems **on TWO sides**. The exam is worth a total of 50 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

You should **KEEP** this copy of your exam.

I will post the solutions on my website sometime after 3:15 today.

1. (6 points) Let $A$ be a fixed $n \times n$ matrix and let $W = \{x \in \mathbb{R}^n \mid Ax = 2x\}$. Is $W$ a vector space? If yes, explain why. If no, give an example to show that the rules of vector space do not hold.

2. (6 points) Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \middle| \, |x_1| = |x_2| \right\}$. Is $W$ a vector space? If yes, explain why. If no, give an example to show that the rules of vector space do not hold.

3. (6 points ) Define "null space". Use complete sentences. Include everything that is necessary, but nothing more.

4. (6 points) Define "non-singular". Use complete sentences. Include everything that is necessary, but nothing more.

5. (6 points) Let $A$ be an $n \times n$ matrix. List three statements that are equivalent to the statement "$A$ is non-singular". Do not repeat your answer to problem 4.

6. (5 points) Let $A$ and $B$ be symmetric $n \times n$ matrices. Does the matrix $AB$ HAVE to be symmetric? If yes, PROVE the statement. If no, give an EXAMPLE.

7. (5 points) Let $v_1, v_2, v_3, v_4$ be vectors in $\mathbb{R}^5$. Suppose that $v_1, v_2, v_3, v_4$ are linearly independent. Do the vectors $v_1, v_2, v_3$ HAVE to be linearly independent? If yes, PROVE the result. If no, show an EXAMPLE.

8. (5 points) Let $v_1$, $v_2$, and $v_3$ be non-zero vectors in $\mathbb{R}^4$. Suppose that $v_i^{\mathrm{T}} v_j = 0$ for all subscripts $i$ and $j$ with $i \neq j$. Prove that $v_1$, $v_2$, and $v_3$ are linearly independent.

9. (5 points) Consider the vectors

$$w = \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}?$$

Is the vector $w$ in the span of the vectors $v_1$, $v_2$, and $v_3$? **Explain thoroughly.**