

Math 544, Exam 2, Summer 2007

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

Please leave room in the upper left corner for the staple.

There are **9** problems **on TWO sides**. The exam is worth a total of 50 points. **SHOW** your work. *CIRCLE* your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail.**

You should **KEEP** this copy of your exam.

I will post the solutions on my website sometime after 3:15 today.

1. (6 points) Let A be a fixed $n \times n$ matrix and let $W = \{x \in \mathbb{R}^n \mid Ax = 2x\}$. Is W a vector space? If yes, explain why. If no, give an example to show that the rules of vector space do not hold.
2. (6 points) Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \mid |x_1| = |x_2| \right\}$. Is W a vector space? If yes, explain why. If no, give an example to show that the rules of vector space do not hold.
3. (6 points) Define "null space". Use complete sentences. Include everything that is necessary, but nothing more.
4. (6 points) Define "non-singular". Use complete sentences. Include everything that is necessary, but nothing more.
5. (6 points) Let A be an $n \times n$ matrix. List three statements that are equivalent to the statement " A is non-singular". Do not repeat your answer to problem 4.
6. (5 points) Let A and B be symmetric $n \times n$ matrices. Does the matrix AB HAVE to be symmetric? If yes, **PROVE** the statement. If no, give an **EXAMPLE**.

7. (5 points) Let v_1, v_2, v_3, v_4 be vectors in \mathbb{R}^5 . Suppose that v_1, v_2, v_3, v_4 are linearly independent. Do the vectors v_1, v_2, v_3 HAVE to be linearly independent? If yes, PROVE the result. If no, show an EXAMPLE.
8. (5 points) Let v_1, v_2 , and v_3 be non-zero vectors in \mathbb{R}^4 . Suppose that $v_i^T v_j = 0$ for all subscripts i and j with $i \neq j$. Prove that v_1, v_2 , and v_3 are linearly independent.
9. (5 points) Consider the vectors

$$w = \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}, \quad v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}?$$

Is the vector w in the span of the vectors v_1, v_2 , and v_3 ? **Explain thoroughly.**