Math 544, Exam 1, Summer 2007

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

Please leave room in the upper left corner for the staple.

There are 7 problems. The exam is worth a total of 50 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website sometime after 3:15 today.

1. (7 points) Define "lineraly independent". Use complete sentences. Include everything that is necessary, but nothing more.

2. (7 points) Define "non-singular". Use complete sentences. Include everything that is necessary, but nothing more.

3. (7 points) Let A be an $n \times n$ matrix. List two statements that are equivalent to the statement " A is non-singular". Do not repeat your answer to problem 2.

4. (8 points) Find the GENERAL solution of the following system of linear equations. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$\begin{array}{rrrrrcl} x_1 & +3x_2 & +4x_3 & +2x_4 & +4x_5 & = & 16 \\ x_1 & +3x_2 & +4x_3 & +3x_4 & +6x_5 & = & 21 \\ 2x_1 & +6x_2 & +8x_3 & +5x_4 & +10x_5 & = & 37 \end{array}$$

5. (7 points) Consider the system of linear equations.

$$\begin{aligned} x_1 + 4ax_2 &= 4 \\ ax_1 + \quad x_2 &= 2. \end{aligned}$$

   (a) Which values for a cause the system to have no solution?
   (b) Which values for a cause the system to have exactly one solution?
   (c) Which values for a cause the system to have an infinite number of solutions?

Explain thoroughly.

6. (7 points) Are the vectors

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

linearly independent? **Explain thoroughly.**

7. (7 points) Let v_1, v_2, v_3, v_4 be vectors in $\mathbb{R}^5$. Suppose that v_1, v_2, v_3, v_4 are linearly dependent. Do the vectors v_1, v_2, v_3 HAVE to be linearly dependent? If yes, PROVE the result. If no, show an EXAMPLE.