Math 544, Final Exam, Summer 2006

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. **Leave room on the upper left hand corner of each page for the staple.** Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 12 problems. The exam is worth a total of 100 points.

SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**. Otherwise, get your grade from VIP.

I will post the solutions on my website shortly after class is finished.

1. (7 points) Define "span". Use complete sentences. Include everything that is necessary, but nothing more.

2. (7 points) Define "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

3. (10 points) Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 & 10 \\ 1 & 2 & -1 & 2 & 16 \\ 2 & 4 & -2 & 3 & 26 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}.$$

   (a) Find the GENERAL solution of the system of linear equations $Ax = b$.
   (b) List three SPECIFIC solutions of $Ax = b$.
   (c) CHECK that the specific solutions satisfy the equations.
   (d) Find a basis for the null space of A.
   (e) Find a basis for the column space of A.
   (f) Find a basis for the row space of A.
   (g) Express each column of A in terms of your answer to (e).
   (h) Express each row of A in terms of your answer to (f).

4. (8 points) Find an orthogonal basis for the null space of $A = [1 \quad 3 \quad 4 \quad 5]$.

5. (8 points) Let $A = \begin{bmatrix} -1 & -10 \\ 5 & 14 \end{bmatrix}$. Find a matrix B with $B^2 = A$. **CHECK your answer.**

6. (10 points) State the four Theorems about vector space dimension.

7. (10 points) Let A be an $n \times n$ matrix. Record eight statements that are equivalent to "the matrix A is invertible".

8. (7 points) Let V be a subspace of $\mathbb{R}^4$. Suppose v_1, v_2, v_3 are linearly independent elements of V. Suppose also that $V \neq \mathbb{R}^4$. Does v_1, v_2, v_3 have to be a basis for V? If yes, prove your assertion. If no, give a counter example.

9. (8 points) Let V be the vector space of symmetric 3×3 matrices. Give a basis for V. Explain your answer.

10. (8 points) Let V be the set of non-singular 2×2 matrices. Is V a vector space? Explain your answer.

11. (7 points) Let $\mathcal{C}$ be the vector space of integrable functions $f\colon \mathbb{R} \to \mathbb{R}$, and let $F\colon \mathcal{C} \to \mathbb{R}$ be the function which is defined by $F(f) = \int_1^2 f(x)dx$ for each $f \in \mathcal{C}$. Is F a linear transformation? Explain your answer.

12. (10 points) In this problem, if M is a matrix, then let $\mathcal{N}(M)$ be the null space of M. Let A and B be $n \times n$ matrices. For each question: if the answer is yes, then prove the statement; if the answer is no, then give a counter example.
    (a) Does $\mathcal{N}(B)$ have to be a subset of $\mathcal{N}(AB)$?
    (b) Does $\mathcal{N}(AB)$ have to be a subset of $\mathcal{N}(B)$?
    (c) Suppose B is non-singular. Does $\mathcal{N}(B)$ have to be a subset of $\mathcal{N}(AB)$?
    (d) Suppose B is non-singular. Does $\mathcal{N}(AB)$ have to be a subset of $\mathcal{N}(B)$?
    (e) Suppose A is non-singular. Does $\mathcal{N}(B)$ have to be a subset of $\mathcal{N}(AB)$?
    (f) Suppose A is non-singular. Does $\mathcal{N}(AB)$ have to be a subset of $\mathcal{N}(B)$?