Math 544, Exam 2, Summer 2006

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. **Leave room on the upper left hand corner of each page for the staple.** Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

The exam is worth a total of 50 points.

SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. **If you are interested, be sure to tell me.**

I will post the solutions on my website shortly after class is finished.

1. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 & 3 \\ 2 & 4 & 6 & 2 & 1 & 5 \\ 2 & 4 & 6 & 1 & 2 & 5 \\ 2 & 4 & 6 & 1 & 1 & 4 \end{bmatrix}.$$

   (a) Find a basis for the null space of A.
   (b) Find a basis for the column space of A.
   (c) Find a basis for the row space of A.
   (d) Express each column of A in terms of your answer to (b).
   (e) Express each row of A in terms of your answer to (c).

2. (8 points) Find an orthogonal basis for the null space of $A = [1 \quad 1 \quad 1 \quad 2]$.

3. (8 points) Let A and B be $n \times n$ matrices with A non-singular. For each question: If the answer is yes, then **PROVE** the assertion. If the answer is no, then give a **COUNTER EXAMPLE**.
   (a) Does the null space of BA have to equal the null space of B?

(b) Does the column space of BA have to equal the column space of B?
(c) Does the dimension of the null space of BA have to equal the dimension of the null space of B?
(d) Does the the dimension of column space of BA have to equal the the dimension of column space of B?

4. (3 points) Define "null space". Use complete sentences. Include everything that is necessary, but nothing more.

5. (3 points) Define "column space". Use complete sentences. Include everything that is necessary, but nothing more.

6. (3 points) Define "dimension". Use complete sentences. Include everything that is necessary, but nothing more.

7. (5 points) Suppose $U \subseteq V$ are vector spaces with the same finite dimension. Does U have to equal V? If the answer is yes, then **PROVE** the assertion. If the answer is no, then give a **COUNTER EXAMPLE**.

8. (5 points) Let v_1, v_2, v_3 be linearly independent vectors in $\mathbb{R}^3$. Can every vector in $\mathbb{R}^3$ be written in terms of v_1, v_2, v_3 in a unique way? If the answer is yes, then **PROVE** the assertion. If the answer is no, then give a **COUNTER EXAMPLE**.

9. (5 points) Let v_1, v_2, v_3, v_4 be vectors in $\mathbb{R}^4$. Suppose that v_1, v_2, v_3 are linearly independent; v_1, v_2, v_4 are linearly independent; v_1, v_3, v_4 are linearly independent; and v_2, v_3, v_4 are linearly independent. Do v_1, v_2, v_3, v_4 have to be linearly independent? If the answer is yes, then **PROVE** the assertion. If the answer is no, then give a **COUNTER EXAMPLE**.