Math 544, Exam 1, Summer 2006

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. **Leave room on the upper left hand corner of each page for the staple.** Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

The exam is worth a total of 50 points. There are 7 problems. Problem one is worth 8 points. Each of the other problems is worth 7 points.

SHOW your work. *CIRCLE* your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. **If you are interested, be sure to tell me.**

I will post the solutions on my website shortly after class is finished.

1. Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 4 & 5 & 1 & 8 \\ 1 & 4 & 5 & 2 & 10 \\ 3 & 12 & 15 & 4 & 26 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix}.$$

2. Consider the system of linear equations.

$$\begin{aligned} x_1 + ax_2 &= 1 \\ ax_1 + 4x_2 &= 2. \end{aligned}$$

   (a) Which values for a cause the system to have no solution?
   (b) Which values for a cause the system to have exactly one solution?
   (c) Which values for a cause the system to have an infinite number of solutions?

Explain thoroughly.

3. Let A and B be 2×2 matrices with AB invertible. Does A have to be invertible? If yes, prove your answer. If no, give a counterexample.

4. Recall that the matrix A is *symmetric* if $A^{\mathrm{T}} = A$. Let A and B be 2×2 symmetric matrices with $AB = BA$. Does AB have to be symmetric? If yes, prove your answer. If no, give a counterexample.

5. Let v_1, v_2, v_3 be linearly independent vectors in $\mathbb{R}^3$. Can every vector in $\mathbb{R}^3$ be written in terms of v_1, v_2, v_3 in a unique way? If yes, prove your answer. If no, give a counterexample.

6. Let A and B be 2×2 matrices with A not equal to the zero matrix and $BA = A^2$. Does B have to equal A? If yes, prove your answer. If no, give a counterexample.

7. Let v_1, v_2, v_3 be vectors in $\mathbb{R}^3$. Suppose that v_1 and v_2 are linearly independent; v_1 and v_3 are linearly independent; and v_2 and v_3 are linearly independent. Do v_1, v_2, v_3 have to be linearly independent? If yes, prove your answer. If no, give a counterexample.