Math 544, Exam 3, Summer 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 7 problems. Problem 1 is worth 14 points. Each of the other problems is worth 6 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. **If you are interested, be sure to tell me.**

I will post the solutions on my website shortly after the class is finished.

1. Let A be the matrix

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 & 4 \\ 1 & 3 & 4 & 3 & 6 \\ 2 & 6 & 8 & 5 & 10 \end{bmatrix}$$

   (a) Find a basis for the null space of A.
   (b) Find a basis for the column space of A.
   (c) Find a basis for the row space of A.
   (d) Write each column of A as a linear combination of your answer to (b).
   (e) Write each row of A as a linear combination of your answer to (c).

2. Let $U \subseteq V$ be subspaces of $\mathbb{R}^n$ with $\dim U = \dim V$. Do U and V HAVE to be equal? If yes, prove your answer. If no, give an example.

3. Let A and B be $n \times n$ matrices. Does the null space of AB HAVE to be a subset of the null space of A? If yes, prove your answer. If no, give an example.

4. Define "null space". Use complete sentences. Include everything that is necessary, but nothing more.

5. Define "dimension". Use complete sentences. Include everything that is necessary, but nothing more.

6. Let

$$V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, 2x_1 + 3x_3 - 4x_3 = 5 \right\}.$$

Is V a vector space? Explain thoroughly.

7. Let $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ be fixed elements of $\mathbb{R}^3$, and let

$$V = \left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, a^{\mathrm{T}} x = 0 \text{ and } b^{\mathrm{T}} x = 0 \right\}.$$

Is V a vector space? Explain thoroughly.