Math 544, Exam 2, Summer 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 7 problems. Problem 1 is worth 8 points. Each of the other problems is worth 7 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your graded exam outside my office door. You may pick it up any time before the next class. **If you are interested, be sure to tell me.**

I will post the solutions on my website shortly after the class is finished.

1. Consider the system of linear equations.

$$\begin{aligned} x_1 + \quad\quad ax_2 &= 1 \\ ax_1 + (3a-2)x_2 &= 2. \end{aligned}$$

   (a) Find all values of a which cause the system to have no solution?
   (b) Find all values of a which cause the system to have exactly one solution?
   (c) Find all values of a which cause the system to have an infinite number of solutions?

Explain thoroughly.

2. Define "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

3. Define "linear combination". Use complete sentences. Include everything that is necessary, but nothing more.

4. Let A be an $n \times n$ matrix. List three statements that are equivalent to " A is non-singular".

5. Let A and B be $n \times n$ matrices with AB equal to the identity matrix. PROVE BA is equal to the identity matrix. ("We did this in class" is not a satisfactory answer. I expect a complete, coherent proof. You are allowed to use any relevant part of problem 4.)

6. Find the general solution of the following system of linear equations.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + x_4 &= 2 \\ x_1 + 2x_2 + 4x_3 + 2x_4 &= 3. \end{aligned}$$

   Also find **three** particular solutions of this system of equations. **Be sure to check** that all three of your particular solutions really satisfy the original system of linear equations.

7. Let v_1, v_2, and v_3 be non-zero vectors in $\mathbb{R}^4$. Suppose that $v_i^{\mathrm{T}} v_j = 0$ for all subscripts i and j with $i \neq j$. Prove that v_1, v_2, and v_3 are linearly independent.