Math 544, Exam 4, Summer 2004

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problems in any order that suits you.

There are 6 problems. Problems 1 and 2 are worth 9 points each. Each of the other problems is worth 8 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office door by noon tomorrow, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Express $v = \begin{bmatrix} 8 \\ 9 \\ 10 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$. (It might be helpful to notice that v_1, v_2 and v_3 are an orthogonal set.)

2. Express $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ as a linear combination of $M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, $M_3 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, and $M_4 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

3. List a basis for the vector space of 3×3 skew symmetric matrices. (No proof is needed.) Recall that the matrix M is skew-symmetric if $M^{\mathrm{T}} = -M$.

4. Let W be the set of all continuous functions $f(x)$ with the property that $\int_0^1 f(x)dx = 0$. Is W a vector space? Explain.

5. Let W be the set of all twice differentiable functions $f(x)$ with the property that $f''(x) + f(x) = e^x$. Is W a vector space? Explain.

6. Find an orthogonal basis for the null space of $A = [1 \quad 3 \quad 4 \quad 5]$.