Math 544, Exam 3, Summer 2004

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Take enough space for each problem. Turn in your solutions in the order: problem 1, problem 2, ... ; although, by using enough paper, you can do the problem in any order that suits you.

There are 9 problems. Problem 1 is worth 10 points. Each of the other problems is worth 5 points. The exam is worth a total of 50 points. SHOW your work. *CIRCLE* your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office door by noon tomorrow, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Let
$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & 12 \\ 1 & 2 & 2 & 1 & 16 \\ 1 & 2 & 2 & 2 & 21 \\ 3 & 6 & 5 & 4 & 49 \end{bmatrix}.$$
   (a) Find a basis for the null space of A.
   (b) Find a basis for the column space of A.
   (c) Express each column of A as a linear combination of the vectors in your answer to (b).
   (d) What is the dimension of the null space of A?
   (e) What is the dimension of the column space of A?

2. Define "span". Use complete sentences.

3. Define "basis". Use complete sentences.

4. Let U and V be subspaces of $\mathbb{R}^n$ and let
$$W = \{w \in \mathbb{R}^n \mid w = u + v \text{ for some } u \in U \text{ and } v \in V\}.$$
   Prove that W is a subspace of $\mathbb{R}^n$.

5. Let V be the set of all polynomials $f(x)$, such that $f(x)$ has real number coefficients, $f(x)$ has degree at most 4, and $f(1) = 0$. Is V a vector space? Explain fairly thoroughly.

6. Let V be the set of all 2×2 non-singular matrices. Is V a vector space? Explain fairly thoroughly.

7. Let A and B be $n \times n$ matrices. Is the column space of AB always contained in the column space of B? If yes, give a proof. If no, give an example.

8. Let A and B be $n \times n$ matrices. Is the null space of AB always contained in the null space of B? If yes, give a proof. If no, give an example.

9. Let
$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$
   Let V be a subspace of $\mathbb{R}^4$. Suppose that $v_1 \in V$, $v_2 \in V$, $v_3 \in V$, and $v_4 \notin V$. Do you have enough information to determine the dimension of V? Explain thoroughly.