Math 544, Exam 2, Summer 2004

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet; start each computational problem on a **new sheet** of paper; and be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.

There are 10 problems. Each problem is worth 5 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will leave your exam outside my office door by noon tomorrow, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 & 10 \\ 1 & 2 & -1 & 2 & 16 \\ 2 & 4 & -2 & 3 & 26 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}.$$

2. Express $a = \begin{bmatrix} 11 \\ 16 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

3. Are the vectors $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ linearly independent or linearly dependent? **Explain.**

4. The vectors v_1, v_2, and v_3 are linearly independent. Do the vectors $v_1 - v_2$, $v_2 - v_3$, and $v_3 - v_1$ have to be linearly independent? If yes, then **prove** the result. If no, then give an **example**.

5. Let A be an $n \times n$ matrix with the property that $Ax = b$ has a unique solution for every vector b in $\mathbb{R}^n$. Does $A^{\mathrm{T}}x = b$ have to have a unique solution for every vector b in $\mathbb{R}^n$? If yes, then **prove** the result. If no, then give an **example**.

6. Define "linearly independent". Use complete sentences.

7. Define "non-singular". Use complete sentences.

8. State the result about the linear dependence of p vectors in m-space. (I call this the Short Fat Theorem).

9. Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \middle| \begin{array}{r} x_1{+}2x_2{+}3x_3{=}0 \\ 2x_1{+}4x_2{+}6x_3{=}0 \\ x_1{-}7x_2{+}9x_3{=}0 \end{array} \right\}$. Is W a vector space? Explain.

10. Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \middle| 0 \leq x_1 x_2 \right\}$. Is W a vector space? Explain.