

## Exam 1, Math 544, Summer, 2003

PRINT Your Name: \_\_\_\_\_

**Please also write your name on the back of the exam.**

There are 9 problems on 6 pages. Problems 1 through 5 are worth 6 points each. Problems 6 through 9 are worth 5 points each. The exam is worth a total of 50 points. **SHOW** your work.

**CIRCLE** your answer. **CHECK** your answer whenever possible.  
**No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail.**

I will leave your exam outside my office door later today, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the class is finished.

1. Find the **GENERAL** solution of the following system of linear equations. Also, list three **SPECIFIC** solutions, if possible. **CHECK** that the specific solutions satisfy the equations.

$$\begin{aligned}1x_1 + 2x_2 &= 1 \\5x_1 + 8x_2 &= 9 \\3x_1 + 5x_2 &= 5\end{aligned}$$

2. Find the **GENERAL** solution of the following system of linear equations. Also, list three **SPECIFIC** solutions, if possible. **CHECK** that the specific solutions satisfy the equations.

$$\begin{aligned}1x_1 + 2x_2 &= 1 \\5x_1 + 8x_2 &= 9 \\3x_1 + 5x_2 &= 6\end{aligned}$$

3. Find the **GENERAL** solution of the following system of linear equations. Also, list three **SPECIFIC** solutions, if possible.

CHECK that the specific solutions satisfy the equations.

$$\begin{array}{rcccccc} x_1 & +3x_2 & + x_3 & +9x_4 & +2x_5 & = & 7 \\ x_1 & +3x_2 & +2x_3 & +13x_4 & +x_5 & = & 3 \\ x_1 & +3x_2 & +2x_3 & +13x_4 & +2x_5 & = & 6 \end{array}$$

4. How many solutions could a system of 4 linear equations in 3 unknowns have? List ALL of the possibilities. **Explain** your answer.
5. How many solutions could a system of 3 linear equations in 4 unknowns have? List ALL of the possibilities. **Explain** your answer.
6. True or False. (If true, explain why or give a proof. If false, give a counter example.) If  $A$ ,  $B$  are  $2 \times 2$  symmetric matrices, then  $AB$  is a symmetric matrix.
7. True or False. (If true, explain why or give a proof. If false, give a counter example.) If  $A$ ,  $B$  are  $2 \times 2$  matrices, then  $(A - B)(A + B) = A^2 - B^2$ .
8. Give an example of  $2 \times 2$  matrices  $A$  and  $B$  with  $A \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$  and  $B \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ , but  $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ .
9. Consider the system of linear equations.

$$\begin{array}{rcc} 3x_1 & +ax_2 & = & 3 \\ ax_1 & +3x_2 & = & 3. \end{array}$$

Which values for  $a$  cause the system to have no solution? Which values for  $a$  cause the system to have exactly one solution? Which values for  $a$  cause the system to have an infinite number of solutions? **Explain.**