Math 544, Summer 2001, Exam 4

PRINT Your Name:________________________________

There are 10 problems on 5 pages. Each problem is worth 5 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators.

1. Define "basis". Use complete sentences.

2. Define "null space". Use complete sentences.

3. Complete the following definition. The vectors $v_1, v_2, \ldots, v_n$ <u>span</u> the vector space V, if $v_1, v_2, \ldots, v_n$ are in V and

4. Suppose A is an $n \times n$ matrix and $Ax = 0$ has infinitely many solutions. Let b be a vector in $\mathbb{R}^n$.
   (a) Can $Ax = b$ have no solution?
   (b) Can $Ax = b$ have exactly one solution?
   (c) Can $Ax = b$ have infinitely many solutions?
   (d) **EXPLAIN** each answer.

5. Suppose $v_1, \ldots, v_n$ are linearly independent vectors in $\mathbb{R}^n$. Do $v_1, \ldots, v_n$ have to span $\mathbb{R}^n$? **EXPLAIN**.

6. Let $\mathcal{B}$ be the basis $\begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix}$ of $\mathbb{R}^2$. Suppose that x is the vector in $\mathbb{R}^2$ whose coordinate vector with respect to the basis $\mathcal{B}$ is $[x]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$. What is the usual representation of x?

7. Let $\mathcal{B}$ be the basis $\begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ of $\mathbb{R}^2$. Let $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ be a vector in $\mathbb{R}^2$. Find the coordinate vector $[x]_{\mathcal{B}}$ of x with respect to the basis $\mathcal{B}$.

8. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If U and V are subspaces of $\mathbb{R}^2$, then the union $U \cup V$ is also a subspace of $\mathbb{R}^2$.

9. Find a basis for the vector space spanned by
$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \text{and} \quad v_4 = \begin{bmatrix} 3 \\ 2 \\ 8 \\ -3 \end{bmatrix}.$$
   Show your work. Check your answer.

10. Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 & 5 & 0 & 9 \\ 1 & 2 & 1 & 7 & 11 & 0 & 17 \\ 1 & 2 & 0 & 3 & 5 & 1 & 16 \\ 1 & 2 & 0 & 3 & 5 & 1 & 16 \\ 1 & 2 & 0 & 3 & 5 & 0 & 9 \end{bmatrix}$. Find a basis for the null space of A. Find a basis for the column space of A. Show your work. Check your answer.