FINAL Exam, Math 544, Spring, 2003

PRINT Your Name:________________________________

There are 20 problems on 12 pages. Each problem is worth 5 points. The exam is worth a total of 100 points. SHOW your work. *CIRCLE* your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your course grade to you. If I don't already know your e-mail address and you want me to know it, send me an e-mail. Otherwise, get your course grade from VIP.

Recall that $\mathcal{P}_n$ is the vector space of polynomials of degree at most n with real number coefficients.

Recall that the matrix A is *skew-symmetric* if $A^{\mathrm{T}} = -A$.

1. Suppose that $T\colon \mathcal{P}_2 \to \mathcal{P}_4$ is a linear transformation, where $T(1) = x^4$, $T(x+1) = x^3 - 2x$, and $T(x^2+2x+1) = x$. Find $T(x^2 + 5x - 1)$.

2. Let W be the subspace of $\mathcal{P}_4$ which is defined as follows: the polynomial $p(x)$ is in W if and only if $p(1) + p(-1) = 0$ and $p(2) + p(-2) = 0$. Find the dimension of W. Explain.

3. Let W be the set of 2×2 matrices whose trace is zero. Is W a vector space? **If YES, then give a basis for W, no proof is needed. If NO, give an example which shows that W is not closed under addition or scalar multiplication.** Recall that the *trace* of a square matrix is the sum of its diagonal elements.

4. Let W be the set of polynomials $p(x)$ in $\mathcal{P}_3$ with $p(0) = 2$. Is W a vector space? **If YES, then give a basis for W, no proof is needed. If NO, give an example which shows that W is not closed under addition or scalar multiplication.**

5. Let W be the set of 2×2 matrices whose determinant is zero. Is W a vector space? **If YES, then give a basis for W, no proof is needed. If NO, give an example which shows that W is not closed under addition or scalar multiplication.**

6. Let W be the set of polynomials $p(x)$ in $\mathcal{P}_3$ with $p(2) = 0$. Is W a vector space? **If YES, then give a basis for W, no proof is needed. If NO, give an example which shows that W is not closed under addition or scalar multiplication.**

7. Find $\lim_{n \to \infty} A^n$, where $A = \begin{bmatrix} 2 & \frac{3}{2} \\ -1 & -\frac{1}{2} \end{bmatrix}$.

8. Define "linear transformation". Use complete sentences.

9. Define "eigenvector". Use complete sentences.

10. Define "linearly independent". Use complete sentences.

11. Define "non-singular". Use complete sentences.

12. Define "null space". Use complete sentences.

13. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.) If A is a 2×2 skew-symmetric matrix, then A has at least one real eigenvalue.

14. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.) Every 4×4 skew-symmetric matrix is singular.

15. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.) If v_1, v_2, v_3 are linearly independent vectors in the vector space V and $T\colon V \to W$ is a linear transformation of vector spaces, then $T(v_1), T(v_2), T(v_3)$ are linearly independent vectors in the vector space W .

16. Find the general solution of the system of linear equations $Ax = b$. If the system of equations has more than one solution, then list three SPECIFIC solutions. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 1 & 2 \\ 1 & 2 & 2 & 4 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 6 \\ 9 \\ 10 \end{bmatrix}.$$

The same matrix A appears in problems 16, 17, and 18.

17. Find the general solution of the system of linear equations $Ax = b$. If the system of equations has more than one solution, then list three SPECIFIC solutions. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 1 & 2 \\ 1 & 2 & 2 & 4 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 6 \\ 9 \\ 9 \end{bmatrix}.$$

The same matrix A appears in problems 16, 17, and 18.

18. Find bases for the row space, column space, and null space of

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 1 & 2 \\ 1 & 2 & 2 & 4 \end{bmatrix}.$$

The same matrix A appears in problems 16, 17, and 18.

19. Find the general solution of the system of linear equations $Ax = b$. If the system of equations has more than one solution, then list three SPECIFIC solutions. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 7 \\ 1 & 3 & 6 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -6 \\ -16 \\ -13 \end{bmatrix}.$$

20. Find an orthogonal basis for the null space of $A = \begin{bmatrix} 1 & 2 & 3 & 5 \end{bmatrix}$.