Exam 3, Math 544, Spring, 2003

PRINT Your Name:________________________________

Please also write your name on the back of the exam.

There are 9 problems on 6 pages. Problem 7 is worth 10 points. Each of the other problems is worth 5 points. The exam is worth a total of 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, send me an e-mail.

I will leave your exam outside my office door about 6PM today, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the exam is finished.

1. Define "column space". Use complete sentences.

2. Define "null space". Use complete sentences.

3. Define "basis". Use complete sentences.

4. Solve the system of equations $Ax = b$ for

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 4 \\ 3 \\ -2 \end{bmatrix}.$$

   You may do the problem any way you like; however, you might want to notice that the columns of A form an orthogonal set.

5. Let W be the vector space which is spanned by

$$w_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad w_2 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \quad \text{and} \quad w_3 = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 1 \end{bmatrix}.$$

   Find an orthogonal basis for W.

6. Find bases for the column space, the row space, and the null space of the matrix

$$A = \begin{bmatrix} 1 & 4 & 0 & 2 & 0 \\ 1 & 4 & 0 & 2 & 0 \\ 1 & 4 & 1 & 2 & 0 \\ 1 & 4 & 1 & 2 & 0 \\ 1 & 4 & 1 & 2 & 1 \end{bmatrix}.$$

7. Let A and B be an $n \times n$ matrices with A non-singular. True or False. (If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE.)
   (a) The null space of AB is equal to the null space of B.
   (b) The column space of AB is equal to the column space of B.
   (c) The rank of AB is equal to the rank of B.

8. Let a and b be vectors in $\mathbb{R}^4$, and let

$$W = \left\{ v \in \mathbb{R}^4 \mid a^{\mathrm{T}} v = 0 \text{ and } b^{\mathrm{T}} v = 0 \right\}.$$

   Is W a subspace of $\mathbb{R}^4$? If so, prove it. If not, give a counterexample. Any legitimate proof or counterexample will suffice.

9. Let A be a 3×4 matrix with nullity one. Does $Ax = b$ have a solution for all vectors b in $\mathbb{R}^3$? If so, prove it. If not, give a counterexample. Any legitimate proof or counterexample will suffice.