Math 544, Exam 1, Spring 2003

PRINT Your Name:________________________________

Please also write your name on the back of the exam.

There are 10 problems on 6 pages. Each problem is worth 5 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you.

I will leave your exam outside my office door about 6PM today, you may pick it up any time between then and the next class.

I will post the solutions on my website shortly after the exam is finished.

1. Find the general solution of the following system of linear equations. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$\begin{array}{rrl} 2x_1 & +3x_2 = & 4 \\ 4x_1 & +6x_2 = & 7 \end{array}$$

2. Find the GENERAL solution of the following system of linear equations. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$\begin{array}{rrrrcl} x_1 & +3x_2 & & +3x_4 & = & 1 \\ x_1 & +3x_2 & +x_3 & +5x_4 & = & 3 \\ 2x_1 & +6x_2 & +x_3 & +8x_4 & = & 4. \end{array}$$

3. Express $b = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

4. Define "linear combination". Use complete sentences.

5. Define "linearly independent". Use complete sentences.

6. STATE the theorem about linear dependence or independence of p vectors in $\mathbb{R}^m$. (I call this theorem the Short Fat Theorem.)

7. True or False. (If true, explain why or give a proof. If false, give a counter example.) If A, B, and C are 2×2 matrices with $A \neq 0$ and $BA = CA$, then $B = C$.

8. True or False. (If true, explain why or give a proof. If false, give a counter example.) If A, B are 2×2 symmetric matrices, then AB is a symmetric matrix.

9. Which numbers a make $A = \begin{bmatrix} 1 & 2 \\ 2 & a \end{bmatrix}$ non-singular? Explain.

10. Let v_1, v_2, v_3 be linearly independent vectors. PROVE that the vectors $v_1 + v_2$, $v_1 - v_2$, v_3 are also linearly independent.