Math 544, Spring 2002, Exam 4

PRINT Your Name:__________________________________

There are 9 problems on 4 pages. Problem 3 is worth 10 points. Each of the other problems is worth 5 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

1. Define "eigenvalue". Use complete sentences.

2. Define "dimension". Use complete sentences.

3. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 7 & 4 \\ -3 & -1 \end{bmatrix}$.

4. Suppose that A is a square matrix and A^2 is the identity matrix. What are the possible eigenvalues for A? Prove your answer.

5. Give an example of a 3×3 matrix A whose eigenvalues are 0 and 1 such that the eigenspace of A which belongs to $\lambda = 0$ has dimension 1 and the eigenspace of A which belongs to $\lambda = 1$ also has dimension one. The matrix A is to have no other eigenvalues other than 0 and 1.

6. Let V be the set of all 3×3 skew-symmetric matrices. (The matrix A is skew-symmetric if $A^{\mathrm{T}} = -A$.) The set V is a vector space. You do NOT have to prove this. Give a basis for V. NO justification is needed.

7. Let $\mathcal{B}$ be the basis $v_1 = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ -9 \end{bmatrix}$ of $\mathbb{R}^2$ and let x be the vector $x = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$. Find the coordinate vector $[x]_{\mathcal{B}}$ of x with respect to the basis $\mathcal{B}$.

8. Suppose a nonhomogeneous system of nine linear equationsin ten unknowns has a solution for all possible constants on the right side of the equations. Is it possible to find two nonzero solutions of the associated homogeneous system that are not multiples of each other? Explain.

9. Find bases for the null space, the column space, and the row space of

$$A = \begin{bmatrix} 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 3 \\ 1 & 0 & 2 & 4 & 6 \end{bmatrix}.$$