Math 544, Spring 2002, Exam 2

PRINT Your Name:______________________________

There are 10 problems on 5 pages. Each problem is worth 5 points. SHOW your work. *CIRCLE* your answer. **CHECK** your answer whenever possible. **No Calculators.**

1. Find the general solution of the following system of linear equations.

$$\begin{aligned} x_1 + 2x_2 + 2x_3 + x_4 &= 2 \\ x_1 + 2x_2 + 3x_3 + 2x_4 &= 3. \end{aligned}$$

   Also find **three** particular solutions of this system of equations. **Be sure to check** that all three of your particular solutions really satisfy the original system of linear equations.

2. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which leaves the origin fixed and rotates the xy-plane by $\pi/4$ radians counterclockwise. What is the matrix M which has the property that $T(v) = Mv$ for all $v \in \mathbb{R}^2$?

3. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which reflects the xy-plane across the line $y = 2x$. What is the matrix M which has the property that $T(v) = Mv$ for all $v \in \mathbb{R}^2$?

4. True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly independent vectors in $\mathbb{R}^4$.

5. True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly dependent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly dependent vectors in $\mathbb{R}^4$.

6. True or False. (If true, give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^3$, then v_1, v_2, v_3 span $\mathbb{R}^3$.

7. True or False. (If true, give a proof. If false, give a counter example.) If A and B are 2×2 matrices, then $(AB)^{\mathrm{T}} = A^{\mathrm{T}} B^{\mathrm{T}}$.

8. Consider the linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}^3$, which is given by $T(v) = Mv$ for all $v \in \mathbb{R}^4$, where $M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 3 \end{bmatrix}$. Is T one-to-one? Is T onto? Explain your answer.

9. Define "onto". Use complete sentences.

10. Define "linear transformation". Use complete sentences.